NOTES AND CORRESPONDENCE

Further Discussion on Simulation of the Modern Arctic Climate by the NCAR CCM1*

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24 July 1995 and 21 December 1995

In a study of the Arctic simulation capabilities of the NCAR CCM1, Bromwich et al. (1994) make a side comment concerning the moisture budget in CCM2, and conclude that (pg. 1067) "locally the semi-Lagrangian transport scheme (of CCM2) has a much smaller error than the positive moisture fixer scheme (of CCM1), but globally the error is approximately the same in both schemes because the semi-Lagrangian scheme is nonconservative." They draw this conclusion from the error or residual term in their moisture budget calculations over caps poleward of 70°, 60°, and 45°N. Their residuals for these polar caps are 4, 14.2, and 21.8 cm yr^{-1}, respectively. We will show below that these large residual values arise from errors in their approximations and that their conclusion concerning the errors in the transport scheme in CCM2 is incorrect.

For the atmosphere and, except in rare cases, for model simulations, diagnostic flux calculations are only approximations and involve errors themselves. If for no other reason, errors are introduced by sampling. Diagnostic calculations are usually done after the fact and may be based on instantaneous or time-averaged daily, 12-hourly, or 6-hourly sampling, instead of sampling every time step as occurs in a model simulation. In addition, the numerical approximations for the diagnosed transport are often different from those actually employed in the model calculations.

In the following, we first discuss the flux approximations involved in a commonly adopted budget calculation and establish the minimum residual achievable when these approximations are applied to polar cap moisture budgets for the CCM2. We then consider the additional errors introduced by temporal sampling and determine the minimum residual that is obtained with the data archived from the CCM2 control simulations. Finally, we discuss the source of the error in the calculations of Bromwich et al. (1994).

Define the column-integrated water vapor Q by

$$Q = \int \frac{q}{g}\, dp, \qquad (1)$$

where q is the specific humidity, p is pressure, and g is gravity. The tendency equation for Q is

$$\frac{\partial Q}{\partial t} = -\nabla \cdot (\mathbf{V}Q) - (P - E), \qquad (2)$$

where P is the precipitation, E is the evaporation, and the column-integrated flux is given by

$$(\mathbf{V}Q) = \int \frac{\mathbf{V}q}{g}\, dp, \qquad (3)$$

in which $\mathbf{V}$ is the horizontal velocity vector with components (u, v). Note that (2) gives the tendency in flux form for the column-integrated water vapor to which the contribution from the vertical flux integrates to zero. Let an overbar denote an area integral

$$\overline{(\quad)} = a^2 \iint (\quad) \cos\phi\, d\phi\, d\lambda. \qquad (4)$$

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* Byrd Polar Research Center Contribution Number 996.

† The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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In (4) a is the radius of the earth, λ is longitude, and ϕ is latitude. Integrating (2) over a north polar cap delineated by latitude $\phi_{j-1/2}$, the southern edge of the grid boxes corresponding to the grid points at latitude ϕ_j, and converting the area integral of the divergence to a line integral of the flux into the polar cap gives

$$\frac{\partial \bar{Q}}{\partial t} = a \int_0^{2\pi} \left(\int \frac{vq}{g} dp \right)_{j-1/2} \times \cos\phi_{j-1/2} d\lambda - (\bar{P} - \bar{E}). \quad (5)$$

For the hybrid (η) vertical coordinate system of the CCM2 (see Hack et al. 1993 for algorithmic details), the vertical integral of the boundary flux (vq) can be approximated by

$$\int \frac{vq}{g} dp = \sum_{k=1}^{K} \frac{v_k q_k}{g} \Delta p_k \quad (6)$$

and similarly for Q, without the meridional velocity component. The subscript k denotes the discrete vertical levels and runs from 1 to K. The pressure difference across the layer is Δp.

The concept of a grid box and thus the latitude of its edge, $\phi_{j-1/2}$, is alien to spectral transform models and their associated Gaussian grids. We define the latitudes $\phi_{j-1/2}$ in the following way. A common approximation for a north polar cap area integral in spectral transform models is

$$\overline{(\ )}^j = \frac{2\pi a^2}{I} \sum_{i=1}^{I} \sum_{l=j}^{J} (\ )_{i,l} w_l, \quad (7)$$

where I is the number of grid points in longitude, w_l are the Gaussian weights used in the model at latitudes ϕ_l, and the latitude index l runs globally from 1 at the southernmost Gaussian latitude to J at the northernmost. The area that the integral represents is poleward of the grid box edge given by latitude $\phi_{j-1/2}$. We define the box edge to be that latitude for which the analytic formula for the polar cap area, $2\pi a^2(1 - \sin\phi_{j-1/2})$, equals the discrete approximation for the area from (7):

$$\sin\phi_{j-1/2} = 1 - \sum_{l=j}^{J} w_l. \quad (8)$$

Integrating (5) in time from t_1 to t_2, and approximating the longitudinal integral with a sum over I grid points gives

$$\bar{Q}^j_{t_2} - \bar{Q}^j_{t_1} = \int_{t_1}^{t_2} \frac{2\pi a}{I} \sum_{i=1}^{I} \left(\int \frac{vq}{g} dp \right)_{i,j-1/2} \times \cos\phi_{j-1/2} dt - \int_{t_1}^{t_2} (\bar{P}^j - \bar{E}^j) dt. \quad (9)$$

If the time integral is approximated by the sum over N steps of size Δt, the budget residual $R_{j-1/2}$ for the cap poleward of latitude $\phi_{j-1/2}$ is defined by

$$R_{j-1/2} = \sum_{\tau=1}^{N} \frac{2\pi a}{I} \sum_{i=1}^{I} \left(\int \frac{vq}{g} dp \right)^{\tau}_{i,j-1/2} \cos\phi_{j-1/2} \Delta t - \sum_{\tau=1}^{N} (\bar{P}^j - \bar{E}^j)^{\tau} \Delta t - (\bar{Q}^j_{t_2} - \bar{Q}^j_{t_1}). \quad (10)$$

For the following calculations, the fluxes at the grid box edges $\phi_{j-1/2}$ are obtained by linear interpolation between the adjacent Gaussian grid points.

In diagnostic calculations with model-simulated data, nonzero values of the residual R do not necessarily reflect model error. There are only a few special circumstances where the budget calculation can be done exactly after the fact. The residual formula (10) is only an approximation to most models, and errors in the approximations contribute to nonzero residual values. It should be kept in mind that the residual R is not expected to be zero for the CCM2 even if the integrals are approximated by a sum over every time step because the approximation for the flux into the polar cap is not equivalent to the numerical approximations used in CCM2. The flux approximation in (5) is equivalent to a conventional centered finite-difference approximation to the flux form of the equations. The CCM2 uses a semi-Lagrangian approximation to the advective form of the equations with a cubic, monotonic interpolant. In addition, the CCM2 approximations include a computational fixer to ensure global mass conservation. This represents a term neglected in the right-hand side of (10) and thus included in the residual R.

We first consider the residual (10) when the temporal integration is sampled every time step. This represents the best that can be done with the approximations of (10). We will then determine the residual for more practical situations corresponding to budget calculations that would be performed from model history data. These indicate the magnitude of the residual expected from diagnostic budget calculations. Figure 1 shows the budget residuals for polar caps. The abscissa is the boundary latitude of the caps. Northern latitudes indicate north polar caps and southern latitudes indicate

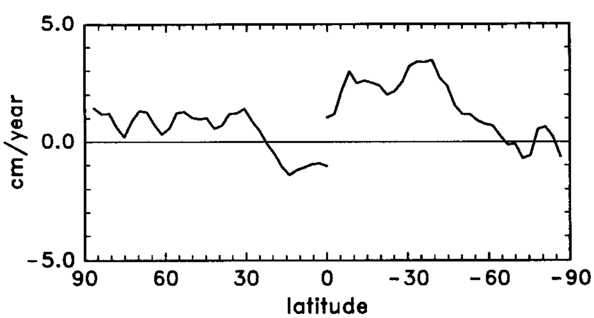

FIG. 1. Budget residuals (cm yr^{-1}) for polar caps from sampling every time step. The abscissa is the boundary latitude of the caps, with northern latitudes indicating north polar caps and southern latitudes indicating south polar caps.

south polar caps. The units are normalized to area averages (cm yr^{-1}) to be consistent with Bromwich et al.(1994). The budget was calculated for one January only, so the difference in the atmospheric reservoir ($\bar{Q}_{t_2} - \bar{Q}_{t_1}$) is included. For annual averages this term can be dropped. In the Northern Hemisphere, the residual is 1 to 2 cm yr^{-1} and in the Southern Hemisphere it is somewhat larger, peaking around 3.5 cm yr^{-1} for caps poleward of 30° to 40°S. These values are significantly smaller than the 21.8 cm yr^{-1} reported by Bromwich et al. (1994). For reference we note that the global average change made by the water vapor fixer in CCM2 is around 3.5 cm yr^{-1} (Williamson and Rasch 1994). Considering that the fixer maximizes in the equatorial region, the residuals calculated above are not inconsistent with the fixer value, although they are not entirely attributable to the fixer as discussed above.

Figure 2 shows the budget residuals calculated from daily, 12-hourly, and 6-hourly averaged data, that is, averages of v, q, and dp. Sampling with daily averages (solid line) increases the residual, particularly in midlatitudes where synoptic and diurnal signals are more important. Calculations with 12- and 6-hourly averages (dashed and dotted curves in Fig. 2, respectively) result in residuals the size of those in Fig. 1. Figure 3 shows the residuals from instantaneous sampling. Daily instantaneous sampling (at 0000 UTC) results in significantly larger residuals as the polar caps extend into the Tropics, with largest values of -10 cm yr^{-1} for hemispheric caps, arising from the introduction of diurnal biases. The residual values are reduced with 12-hourly instantaneous sampling, to values around 3 and 4 cm yr^{-1} in Northern and Southern Hemisphere caps bounded in midlatitudes and -4 cm yr^{-1} for Northern Hemispheric caps bounded by the Tropics. Six-hourly instantaneous sampling results in residuals comparable to those in Fig. 1.

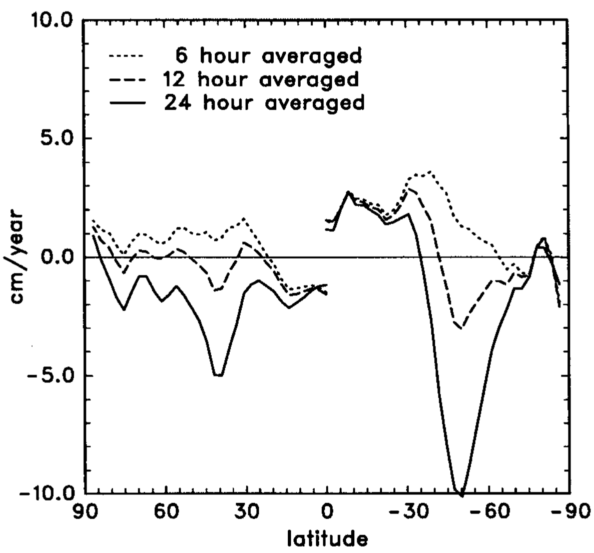

FIG. 2. Budget residuals (cm yr^{-1}) for polar caps from daily, 12-hourly, and 6-hourly averaged data. See Fig. 1.

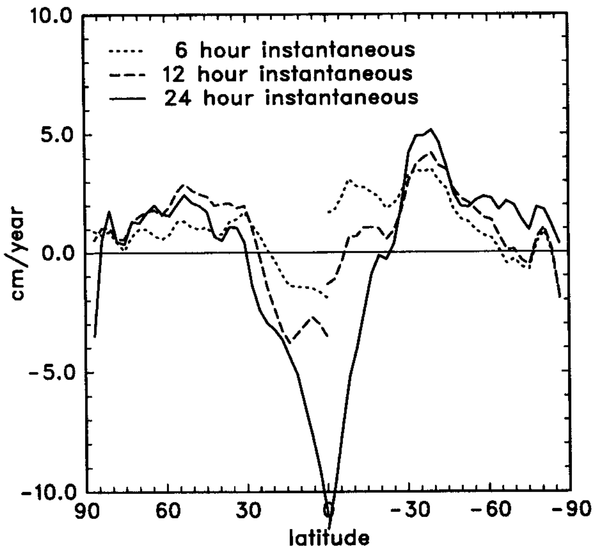

FIG. 3. Budget residuals (cm yr^{-1}) for polar caps from daily (0000 UTC), 12-hourly (0000 and 1200 UTC), and 6-hourly (0000, 0600, 1200, and 1800 UTC) instantaneous data. See Fig. 1.

The control simulations with CCM2, which were available to Bromwich et al. (1994), archived daily averaged data (Williamson 1993). Thus, the budget residuals calculated from daily averaged data in Fig. 2 (solid line) represent the level of residual expected from such data. The residuals in Fig. 2 are for a single January. A similar calculation for 1 year from the CCM2 control run results in a similar graph. With daily averages, the temporal aliasing produces a larger residual in midlatitudes than with instantaneous data, but it is still well below the values of Bromwich et al. (1994). Their values at 45°N are four times larger and therefore must be due to additional approximation errors in their calculations. In fact, they did not use a flux integral into the polar cap as in (5), but rather averaged an approximation to the horizontal advection over the polar cap (R.-Y. Tzeng 1995, personal communication)

$$\nabla\cdot(\mathbf{V}Q) \approx \int \left(\frac{u}{a\cos\phi}\frac{\partial q}{\partial \lambda} + \frac{v}{a}\frac{\partial q}{\partial \phi}\right)\frac{dp}{g} \qquad (11)$$

with the derivatives approximated by centered finite differences, and neglected the vertical advection and atmospheric mass convergence. Unlike the flux form, the vertical advection does not integrate to zero through the vertical column and its omission is the major source of error in their calculation. This was independently verified by the second author using analyses from the European Centre for Medium-Range Weather Forecasts for 1986. Notice that the moisture budget deficiencies outlined here have a small impact in the area poleward of 70°N, the focus of the Bromwich et al. (1994) study.

In summary, budget calculations for model simulations must be carefully done, but even then often represent only an approximation to the actual prediction equations of the model. In addition, the temporal sampling must be taken into consideration when studying budget residuals. With the CCM2, which is based on a monotonic, semi-Lagrangian advection scheme for water vapor, flux-based residuals for the moisture budget over polar caps are on the order of 1 to 4 cm yr^{-1} when the sampling is done every time step. Daily instantaneous or averaged sampling results in residuals ranging from 5 to 10 cm yr^{-1}, while 6- and 12-hourly instantaneous or averaged sampling results in residuals comparable to those from sampling every time step. With daily averaged sampling as in CCM2, the flux terms themselves (vq and uq) should be archived along with the individual values (v, u, and q).

Acknowledgments. We would like to thank James Hack, Philip Rasch, and an anonymous reviewer for helpful comments on the original draft, and Paula Drager for editorial improvements. The second author's contribution was supported by National Science Foundation Grant OPP-9224184.

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